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G. von Gehlen<sup>(x)</sup>: ON THE POSSIBILITY OF RESONANT INTERACTIONS OF THE MUON.

The explanation of the mass difference between the muon and the electron is still one of the major unsolved problems in elementary particle physics. One would like to ascribe this mass difference to some interaction of the muon which is not shared by the electron. Various experiments have been proposed in order to detect anomalous interactions in muon electrodynamics, but as yet no such anomaly has been found. As has been emphasized by some people, a resonant interaction which is strong only in a narrow energy interval would be difficult to detect in the usual experiments.

If we assume that the number of muons and muon-neutrinos (minus the number of their antiparticles) is conserved, we may classify the possible resonant states which interact with the muon according to their muon number  $N_\mu$ .

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We start with  $N_\mu = 0$ . This state must be coupled to at least two muon lines and should contribute to muon bremsstrahlung and muon pair production. Experiments which measure the cross section for muon pair production as a function of the center-of-mass energy of the two muons are planned at Frascati<sup>(1)</sup>. In colliding beam experiments this resonance should show up in lowest order of  $e^2$ , if it has the same quantum numbers as the photon. The case of  $N_\mu = 2$  is outside the possibilities of present experiments, in order to investigate this case, one has to study for instance  $\mu$  - trident production.

Perhaps the most interesting case is  $N_\mu = 1$ . Here we may combine the muon with every other elementary particle except the  $\nu_\mu$ . As an example we consider the possibility of an excited state of the muon which can be reached in  $\pi - \mu$  - scattering. If the resonance energy is high enough, it does not appreciably influence  $\pi \rightarrow \mu + \nu$  decay. Instead it should influence the neutrino spectrum in  $K_{\mu 3}$  - decay and it should increase the number of large-angle scattered muons in pion production in muon-nucleon collisions. In the latter process the usual electromagnetic mechanism favours small angle inelastic muon scattering in contrast to a mechanism which proceeds through the formation of a  $\pi - \mu$  bound state as in Fig. 1. Of course there should also be an anomaly in nuclear scattering of muons

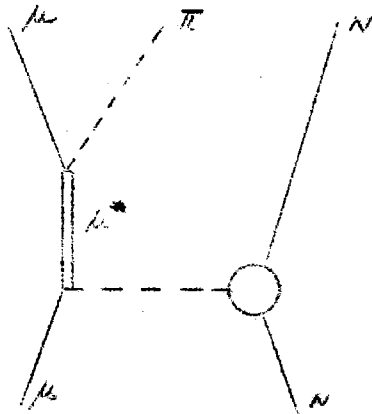


FIG. 1

and in  $\mu$  - pair production, but in the differential cross section for these processes there appears only an integral over the  $\pi - \mu$  interaction. We now want to show that the great precision of the  $g-2$  experiment<sup>(2)</sup> which puts an upper limit of  $10^{-5} e/2m_\mu$  to non-electromagnetic contributions to the muon magnetic moment already allows to exclude to a large degree the existence of a resonant  $\pi - \mu$  interaction.

We assume that the muon-pion interaction is strong only in a single angular momentum state. In order to fix ideas, the cross section for  $\mu - \pi$  scattering is taken to be zero outside the resonance region. In the resonance region it shall reach the maximum value allowed by unitarity. The exact shape of the cross section is immaterial for our considerations, for simplicity we take it to be square shaped.

If such a  $\mu - \pi$  interaction exists, its T-matrix should enter in an analogous manner in the muon magnetic moment calculations as does the  $N - \pi$  scattering matrix element for the nucleon moment. Therefore our calculation is completely analogous to the work of Frazer and Fulco<sup>(3)</sup>. We assume that there is no Born term but only the resonant scattering contribution, as shown in Fig.2. Diagram Fig.3 is disregarded as well as the fact, that the Legendre expansion in our case fails to converge for  $t' < -16.5 \mu^2$ .

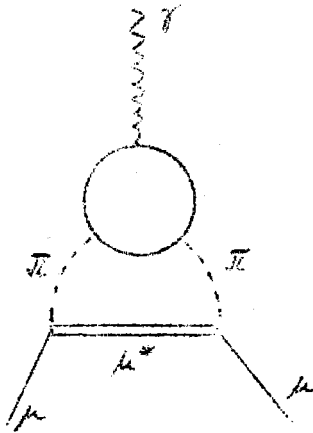


FIG.2

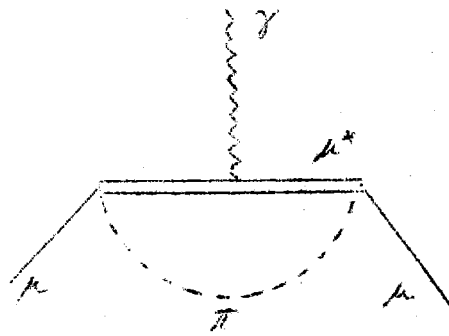


FIG.3

In the case of a resonance with  $l = 1$  and parity  $-1$ , we get from eq. (5.10) of FF1 (using the notations of FF1,  $m$  is now the muon mass,  $\sqrt{s}_R$  the resonance energy and  $\Delta \sqrt{s}$  the resonance width):

$$(1) \quad b_1 = \frac{1}{k(E_S - m)} \Delta s \delta(s - s_R), \quad a_1 = -(\sqrt{s} - m)b_1,$$

which is to be substituted into (2.7b) of FF2. Denoting by  $\beta$  the contribution of the  $\mu - \pi$  - resonance to the magnetic moment of the muon, we have:

$$(2) \quad \beta = G_2^V(0) = \frac{2\sqrt{s}\Delta\sqrt{s}}{4\pi^2 k(E_S - m)} \int_{4\mu^2}^{\infty} dt \frac{e|F_\pi|^2 q^3}{t^{3/2}} \times$$

$$\times \int_{-\infty}^a dt' \frac{1}{(t' - t)F_\pi(t')} \left\{ -\frac{\sqrt{s} - m}{p_-^2 q_-^2} z + \frac{m}{2p_-^3 q_-} (3z^2 - 1) \right\}$$

Analogous expressions result for other choices of the angular momentum of the resonance.

In the work of FF, the upper limit of the  $t'$ -integration,  $a$ , is close to  $+4\mu^2$ . This is not so in our case where we have  $a = -0.161 \mu^2$  for  $\sqrt{s_R} = 250$  MeV, and  $a = -15.7 \mu^2$  for  $\sqrt{s_R} = 600$  MeV. In the first integral we use a  $\delta$ -function for  $|F_\pi(t)|^2$ , putting

$$|F_\pi(t)|^2 = 150\mu^2 \delta(s-s_R),$$

but we are not allowed to set  $F_\pi(t') = 1$  in the second integral, because we need  $F_\pi(t')$  far away from  $t' = 0$ . However, if we use the Clementel-Villi-approximation  $F_\pi(t') = t_R/(t_R-t')$ . It turns out that the integral over  $t'$  vanishes. The contribution of the low- $t'$ -region is exactly compensated by the high- $t'$ -contribution. Therefore any calculation of the magnetic moment is based only on the knowledge of  $F_\pi(t')$  we have, which goes beyond the C-V-approximation, and on the knowledge of deviations from (2), e.g. inelastic contributions(4).

In order to get an estimate of  $\beta$ , we take essentially the same attitude as FF: We assume that the high- $t'$ -contribution is somehow suppressed so that it does not compensate the low  $t'$ -contribution too much. Then the order of magnitude of the whole integral is determined by the low- $t'$ -region. Cutting off the  $t'$ -integration at  $t'_m$ , we find ( $\Delta\sqrt{s}$  is measured in MeV):

$\sqrt{s_R}$ in MeV	$\beta$ in $(e/2m)\Delta\sqrt{s}$	$t'_m$
250	-1300	- 6 $\mu^2$
250	- 250	- 100 $\mu^2$
600	- 3.3	- 40 $\mu^2$
600	- 0.3	-1000 $\mu^2$

If we do not want to destroy the excellent agreement between the purely electromagnetic calculations(5) and the experiment(2),  $\beta$  should not exceed  $10^{-5} e/2m$ . Assuming  $\beta \approx -200\Delta\sqrt{s} e/2m$  for  $\sqrt{s_R} = 250$  MeV and  $\beta \approx -0.5\Delta\sqrt{s} e/2m$  for  $\sqrt{s_R} = 600$  MeV, this means that  $\Delta\sqrt{s} < 0.05$  eV for  $\sqrt{s_R} = 250$  MeV and  $\Delta\sqrt{s} < 20$  eV for  $\sqrt{s_R} = 600$  MeV. Even if our crude

estimate were wrong by an order of magnitude, we still get an upper limit for the resonance width which is less than 1 KeV (the resonance cross section always normalized to the unitary limit). Similar results are expected with other assumptions for the angular momentum and parity of the resonance, but for higher angular momenta the integrals in (2) diverge if we use a C-V-type form factor.

Part of this work was performed while the author was at the Institut für theoretische Kernphysik, Karlsruhe. The author wants to thank Dr. M.A. Spano for performing a numerical calculation on the IBM 1610 computer.

Footnotes:

- (1) - Private communication
- (2) - G.Charpak, F.J.M. Farley, R.L. Garwin, T. Muller, J.C. Sens, and A. Zichichi, Proc. of the 1962 International Conference on High-Energy Physics at CERN, p. 476
- (3) - W.R. Frazer and J.R. Fulco, Phys. Rev. 117, 1603 (1960), Phys. Rev. 117, 1609 (1960), referred to as FF1 and FF2, respectively.
- (4) - This statement applies also to the calculation of the nucleon magnetic moment. Of course the resonance shape used in FF2 is not simply a  $\delta$ -function and the asymptotic behaviour of (2.12) in FF2 is quite different from the C-V-form.
- (5) - A. Petermann, Helv. Phys. Acta 30, 407 (1957); C. Sommerfeld, Phys. Rev. 107, 328 (1957).